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Superposing Semi-Geostrophic Potential Vorticity Anomalies

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Summary

A numerical method for inversion of the semi-geostrophic potential vorticity equation is described. It is used to examine the validity of a strictly linear superposition principle when applied to this high order balance model. These results are important when aiming to extend rigorously the value of recent 'attribution' ideas. Experiments conclude that the non-linear components of the inversion have only a very short-range effect with a magnitude which increases with the magnitude of the potential vorticity anomalies.

For balance of higher-order than quasi-geostrophic these results show that, whilst flow and temperature of an anomaly are non-linear functions of PV magnitude, spatially separated anomalies are quasi-linearly superposable.

Keywords: potential vorticity, semi-geostrophic equations, attribution.

1. Introduction

Recently ideas of 'attribution' have been proposed to describe the individual contributions of various forcing mechanisms in the generation of cyclonic storms ((Davis and Emanuel (1991), Davis (1992), Bishop and Thorpe (1994), Thorpe and Bishop (1995)). It involves dividing the atmosphere in terms of a collection of potential vorticity (PV) anomalies. Each anomaly has associated with it a unique geopotential field analogous to the induction of an electric field, by an electric charge. This field can be obtained by inverting the PV equation for given boundary conditions. The electrostatics analogy and 'action-at-a-distance' have been discussed in Bishop and Thorpe (1994) for quasi-geostrophic PV and extended to the Ertel-Rossby formulation in Thorpe and Bishop (1995).

One of the attractive aspects of quasi-geostrophic PV inversion is its linearity. A consequence of this is that the flow and temperature derived from the geopotential field attributed to individual anomalies are also linear functions, and therefore can be simply added together i.e. a linear superposition principle. It is only under these conditions that the concept of attribution is a linear one. However quasi-geostrophic PV and its inversion have severe limitations particularly as the Rossby number approaches unity. Higher order approximations to PV inherently involve a non-linearity which means that a strictly linear superposition principle will not apply. This is a complication which in principle could limit the usefulness of PV-thinking as applied to real atmospheric flows. However if the consequences of the nonlinear terms can be shown to be small then it may still be a useful concept to explore. Here we assess quantitatively how significant this non-linearity of superposition is for semi-geostrophic PV inversion.

To illustrate the non-linearity in the superposition imagine a given PV anomaly to be composed of the sum of two exactly overlapping anomalies each of half the amplitude. Since the flow and temperature field is not a *linear* function of the magnitude of the anomaly, superposing the solutions will give a different field to that obtained by inverting a single anomaly of the whole amplitude. The non-linearity can be evaluated by performing an inversion for the full anomaly and then subtracting the result of a linear superposition of the inversion of the two sub-anomalies from which we imagine the full anomaly to be composed. In the limit of the two anomalies being removed very far apart, the linear superposition principle would be exact since the fields created by each no longer interact. If the non-linearity decreases rapidly as the distance between the two anomalies increases then the linear superposition principle can, for all practical purposes, be retained for spatially distinct anomalies. It would then only be necessary to account for the nonlinear dependence of the flow and temperature on the PV amplitude of each anomaly alone.

In this note we quantify the superposability by finding the full non-linear inversion solutions for anomalies of varying sizes and magnitudes.

2. Semi-geostrophic analytical solution.

As discussed in Thorpe and Bishop (1995) the semi-geostrophic PV can be written as a function of the geopotential ϕ' in the following way if (and only if) the PV anomaly has a radial symmetry:

$$s' = \frac{1}{f r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi'}{\partial r} + \frac{1}{f^2} r \left(\frac{\partial \phi'}{\partial r}\right)^2 + \frac{1}{3f^4}\left(\frac{\partial \phi'}{\partial r}\right)^3 \right) \tag{1}$$

where s' is the potential vorticity (or more accurately the potential vorticity substance) anomaly suitably normalised to have the dimensions of vorticity and embedded in a motionless infinite atmosphere with constant static stability N^2 and Coriolis parameter f, and $r = \left(x^2 + y^2 + \frac{N^2}{f^2} z^2 \right)^{1/2}$ is a radial coordinate centred on the anomaly.

Solutions for a localised ball (radius r_o) of semi-geostrophic potential vorticity of uniform strength s_o, can be found in terms of the geopotential gradient, $\partial\phi'/\partial r$:

$$\begin{aligned} \frac{\partial \phi'}{\partial r} &= f^2 r \left(\left(1 + \frac{s_o}{f}\frac{r_o^3}{r^3} \right)^{1/3} - 1 \right) \qquad r > r_o \\ \frac{\partial \phi'}{\partial r} &= f^2 r \left(\left(1 + \frac{s_o}{f} \right)^{1/3} - 1 \right) \qquad r < r_o \end{aligned} \tag{2}$$

and analytical solutions for temperature and wind can be found directly from these equations since

$$\begin{aligned} u = -\frac{1}{f}\frac{\partial \phi'}{\partial y}, \quad v = \frac{1}{f}\frac{\partial \phi'}{\partial x}. \\ \theta = \theta' + \bar{\theta} \end{aligned} \tag{3}$$

where θ' is the potential temperature perturbation due to the anomaly and $\bar{\theta}$ is the potential temperature due to the atmospheric lapse rate as given by:

$$\frac{\partial \phi'}{\partial z} = \frac{g\theta'}{\theta_o}, \quad \frac{d\bar{\theta}}{dz} = \frac{\theta_o N^2}{g} \tag{4}$$

The *non-linear* dependence of, say, the θ field on PV magnitude can be seen in:

$$\begin{aligned} \theta &= \theta_o \left(1 + \frac{N^2}{g} z \left(1 + \frac{s_o}{f}\frac{r_o^3}{r^3} \right)^{1/3} \right) \qquad r > r_o \\ \theta &= \theta_o \left(1 + \frac{N^2}{g} z \left(1 + \frac{s_o}{f} \right)^{1/3} \right) \qquad r < r_o \end{aligned} \tag{5}$$

These *analytical* solutions can be used to test the validity of numerical solutions.

3. Semi-geostrophic numerical solution for a single anomaly

To consider superposition of two (or more) anomalies it is necessary to create a more general method of solution for the PV inversion equation i.e. a *numerical* solution. In Cartesian coordinates equation (1) can be written:

$$
\left.
\begin{aligned}
s' = \frac{1}{f}\left(\phi'_{xx} + \phi'_{yy} + \frac{f^2}{N^2}\phi'_{zz}\right) \qquad & \\
+\frac{1}{fN^2}\left(\left(\phi'_{xx} + \phi'_{yy}\right)\phi'_{zz} - \phi'^2_{xz} - \phi'^2_{yz}\right) & \\
+\frac{1}{f^3}\left(\phi'_{xx}\phi'_{yy} - \phi'^2_{xy}\right) & \\
+\frac{1}{f^3N^2}\det\begin{bmatrix} \phi'_{xx} & \phi'_{xy} & \phi'_{xz} \\ \phi'_{xy} & \phi'_{yy} & \phi'_{yz} \\ \phi'_{xz} & \phi'_{yz} & \phi'_{zz} \end{bmatrix} &
\end{aligned}
\right\}
\begin{aligned}
&= Q. - G. \\
&\\
&\\
&\

same height and at an equal distance from the left and right boundaries respectively. The velocity and θ fields produced separately for each were then added together to produce a 'linear superposition inversion' (denoted 3) of the two balls together.

- Secondly the same two anomalies were included in the domain together and a 'complete inversion' (denoted 4) was performed using the numerical method developed.

This experiment was conducted four times, twice as above, and twice with vertical rather than horizontal alignment, for the anomalies. In the first and third experiments the anomalies had equal magnitude $4\text{x}10^{-4}\text{s}^{-1}$, and in the second and fourth experiments the anomalies had equal and *opposite* magnitude $0.9\text{x}10^{-4}\text{s}^{-1}$, and $-0.9\text{x}10^{-4}\text{s}^{-1}$. These results were then compared by producing graphical representations of the flow, velocity field, and potential temperature pattern , and also, by differencing the two sets of fields produced, as below:

Maximum θ difference = $\left|\left(\theta_4-\theta_3\right)_{\text{max.}}\right|$ where $\theta_3=\theta_1+\theta_2-\left(\theta_o\left(1+\frac{N^2}{g}z\right)\right)$,

maximum velocity difference = $\left|\left(v_4-v_3\right)_{\text{max.}}\right|$ where $v_3=v_1+v_2$,

normalised θ difference= $\left|\frac{\left(\theta_4-\theta_3\right)_{\max}}{\theta'_{4\max}}\right|$ where $\theta'_{4\max}=\left(\theta_4-\theta_o\left(1+\frac{N^2}{g}z\right)\right)_{\max}$,and

normalised velocity difference = $\left|\frac{\left(v_4-v_3\right)_{\max}}{v_{4\max}}\right|$

where $(\theta_o(1+N^2z/g)$ is the background potential temperature distribution and this must be subtracted off to obtain the potential temperature anomaly due to the PV anomaly only. Throughout this paper N^2 is assumed constant and equal to $1\text{x}10^{-4}\text{s}^{-2}$, similarly the Coriolis parameter f = $1\text{x}10^{-4}\text{s}^{-1}$).

This procedure was repeated for anomalies at varying distances apart, (as measured by the distance between the centres of the two anomalies) such that the distance apart divided by the diameter of the anomalies decreased from 3 to 0. For overlapping anomalies the 'complete inversion' was performed for an anomaly of $8\text{x}10^{-4}\text{s}^{-1}$ in the regions of overlap and for $4\text{x}10^{-4}\text{s}^{-1}$ otherwise.

Experiments five and six were performed varying the magnitude of the anomalies from $-1\text{x}10^{-4}\text{s}^{-1}$ to $4\text{x}10^{-4}\text{s}^{-1}$ whilst keeping the distance apart horizontally and vertically respectively, constant at 1.5 diameters.

5. Results and Conclusion

The results of these experiments are shown in figures 1 to 4. Figure 1 shows the difference field for potential temperature with anomalies a distance apart equal to 1.5 diameters, radius 200km for the four experimental cases. Figures 1b) and d) show that for one anomaly negative and the other positive the maximum error occurs near or coincident with the negative anomaly. This follows from the greater effect a single negative anomaly has upon the field than a single positive anomaly of the same magnitude. Figures 1 a) and c) show that for both anomalies magnitude 4f the maximum error occurs symmetrically between the PV spheres. For both vertically aligned cases the maximum error is in fact coincident with one or both of the anomalies, rather than between as for the horizontal cases.

An overall picture of the error in linearly superposing anomalies can be seen in figures 2 and 3. Figure 2 shows the maximum error in all four cases, and figure 3 shows the percentage (or normalised) error in the four cases. For spatially distinct anomalies (i.e. distance apart/diameter of anomalies greater than one) the error remains negligible, around 7.5% of the maximum values of the anomalous potential temperature and velocity at most, and this decreases with increasing separation. The rapid spatial decay of the non-linearities is anticipated in Thorpe and Bishop (1995) in their analysis of the spatial dependence

of the semi-geostrophic "dielectric constant". For experiments with s_0=4f, as the anomalies approach each other the error increases rapidly, reaching a maximum error of around 30% for complete overlap. This "overlap" error in linear superposition can be seen, from equation (5), to be due to the non-linear dependence of ϕ' or potential temperature on the magnitude of the PV anomaly. The error is approximately similar in both the velocity and potential temperature fields. For experiments with s_0=0.9f and -0.9f the percentage difference becomes unbounded since the 'complete inversion' has both zero velocity and zero potential temperature perturbation associated with it, and the error in making a linear superposition is the maximum possible. On the other hand figures 3b) and d) show that the error in linear superposition falls off even more rapidly as the anomalies of opposite sign are moved apart. These results have implications when wishing to divide a distinct anomaly into more easily describable or invertible shapes. Similar results were obtained for anomalies of half the radius although percentage errors were slightly reduced.

Figure 4 shows results from experiments changing the magnitude of the anomalies whilst they remain 1.5 diameters apart in both the horizontal and vertical direction. It can be seen that the amplitude has a strong influence on the error, with a near-linear variation in percentage difference so that linear superposition is a better approximation for smaller magnitude anomaly inversions. However for negative anomalies the error increases greatly near to -f , although still remaining below 5%. This dependence can be described analytically for ball-shaped semi-geostrophic PV anomalies, as shown by Thorpe and Bishop (1995).

Quasi-geostrophic equations are widely used to determine the effects of potential vorticity anomalies, and the method of attribution has been developed with this in mind. However there are many situations in which quasi-geostrophic theory is not applicable and a more accurate approximation is required. These results show that attribution ideas can also be extended to semi-geostrophic theory, with a few qualifications. It is clear that the non-linear component of the semi-geostrophic inversion does indeed have a very short-range effect as hypothesised by Thorpe and Bishop (1995).

In this paper we have used idealised PV anomalies so that simple conclusions can be made. These results show that for balance of higher-order than quasi-geostrophic, flow and temperature of an anomaly are non-linear functions of PV magnitude whilst spatially separated anomalies are quasi-linearly superposable. It is important that these conclusions be used to interpret superposition for the more complex structure of PV anomalies observed in the atmosphere.

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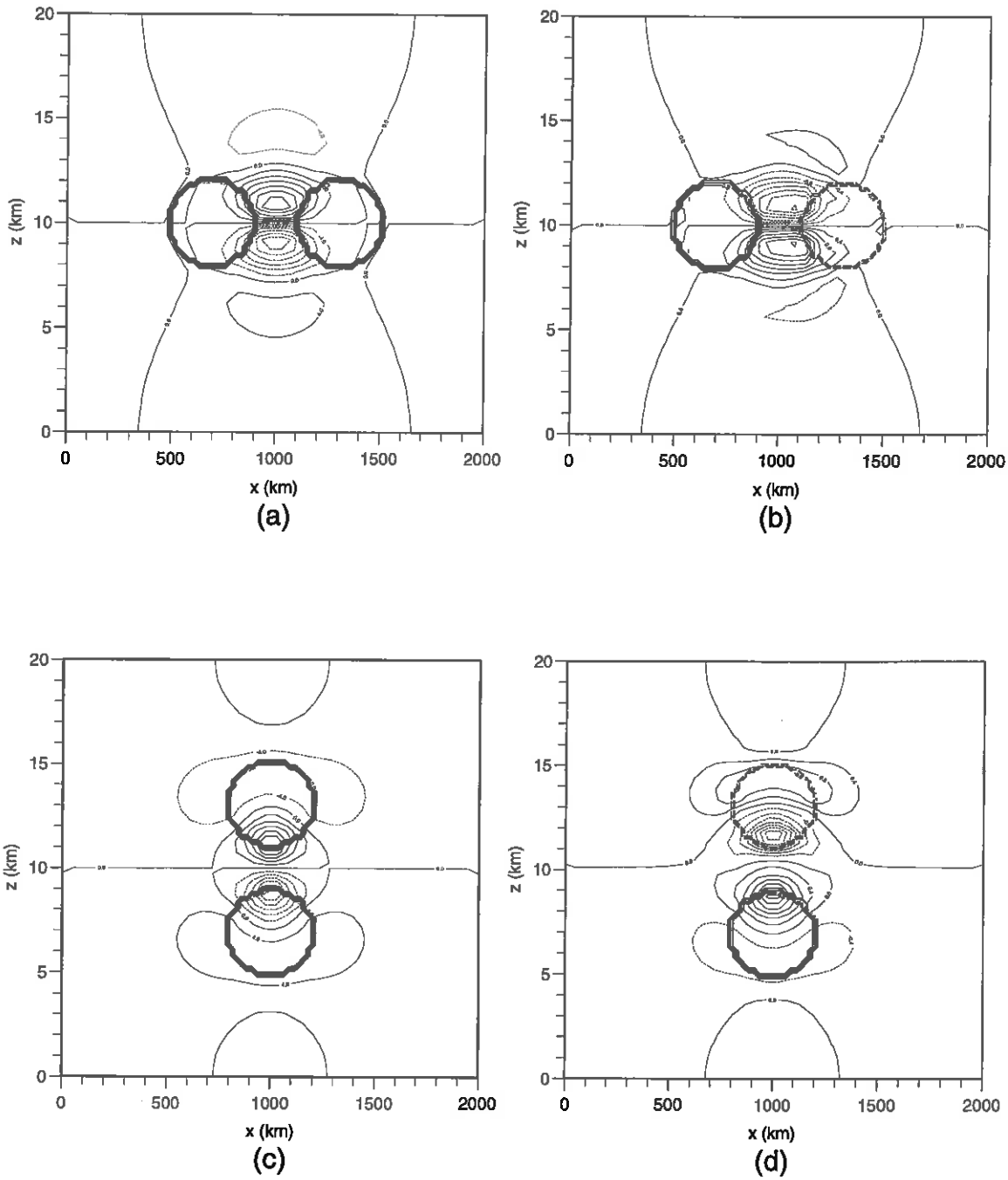

Fig 1 Difference in potential temperature field and full linearly superposed inversion solutions for anomalies whose centres are 1.5 diameters apart a) horizontally aligned and magnitude 4f, b) horizontally aligned and magnitudes 0.9f and -0.9f, c) vertically aligned and magnitude 4f, and d) vertically aligned and magnitudes 0.9f and -0.9f. (Values multiplied by 100, contour interval 0.04°K, negative contours dashed). Position of anomalies also marked in bold lines (Contour interval $0.8 \times 10^{-4} s^{-1}$ for a) and c), $0.4 \times 10^{-4} s^{-1}$ for b) and d)).

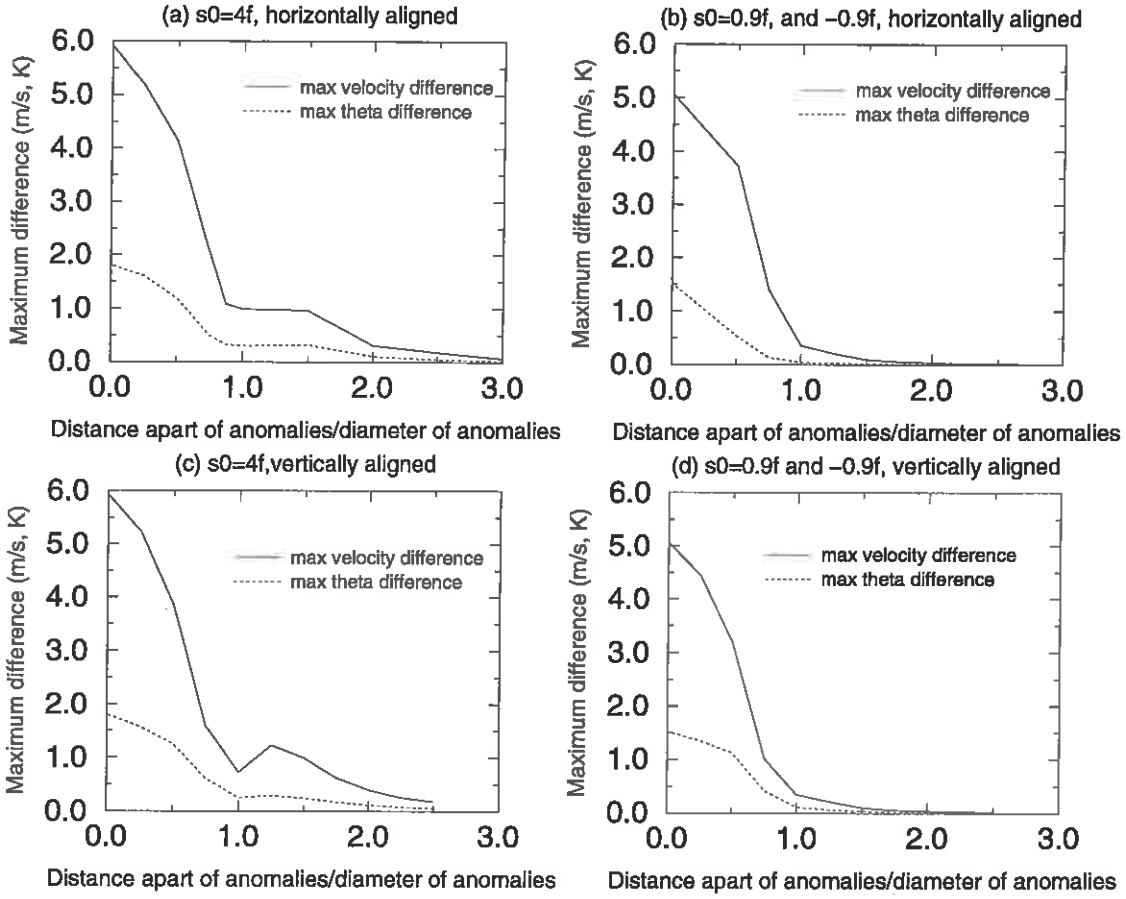

Fig 2: Maximum difference between linearly superposed and full solutions for two anomalies at different distances apart with alignment and magnitude as stated on each graph. Note: the distance apart of the anomalies refers to the distance between the centres of the two anomalies.

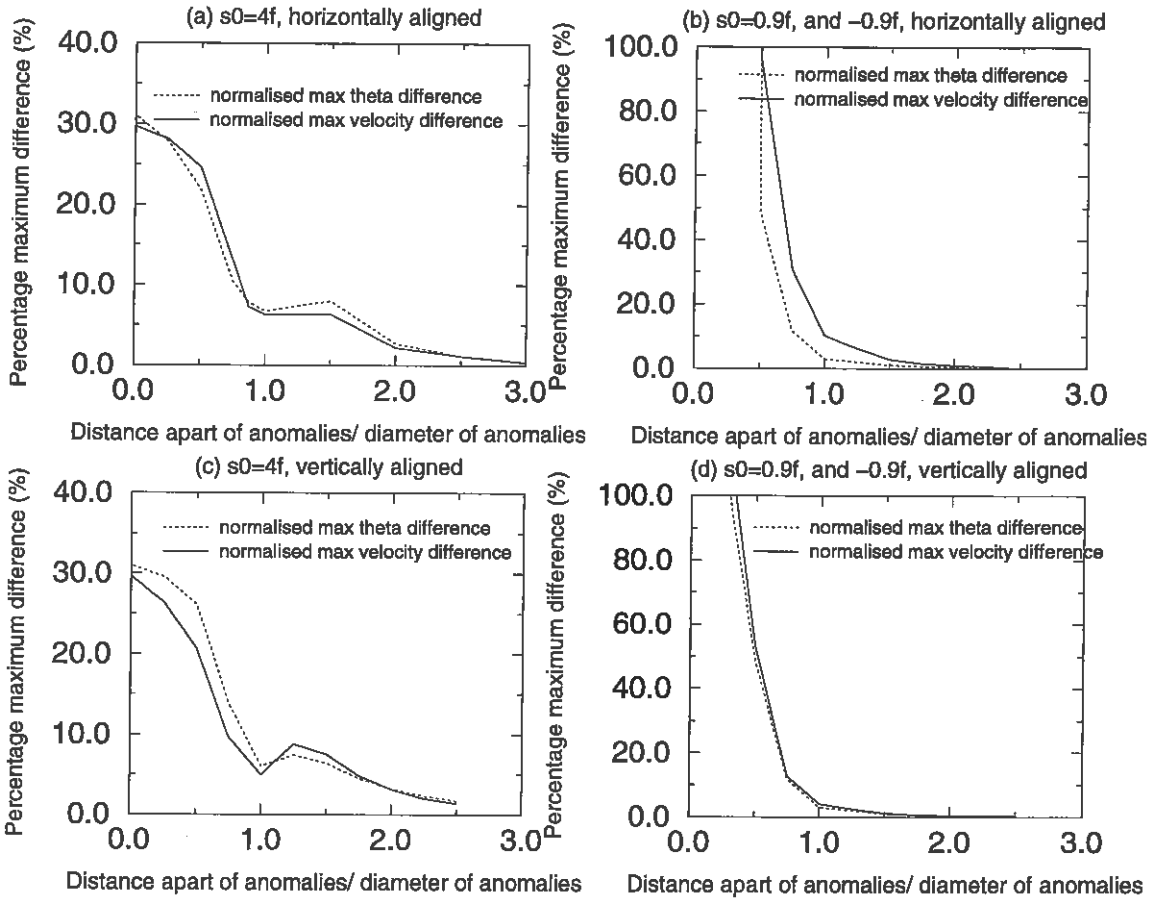

Fig 3: Percentage maximum difference between linearly superposed and full solutions for two anomalies at different distances apart with alignment and magnitude as stated on each graph. Note: the distance apart of the anomalies refers to the distance between the centres of the two anomalies.

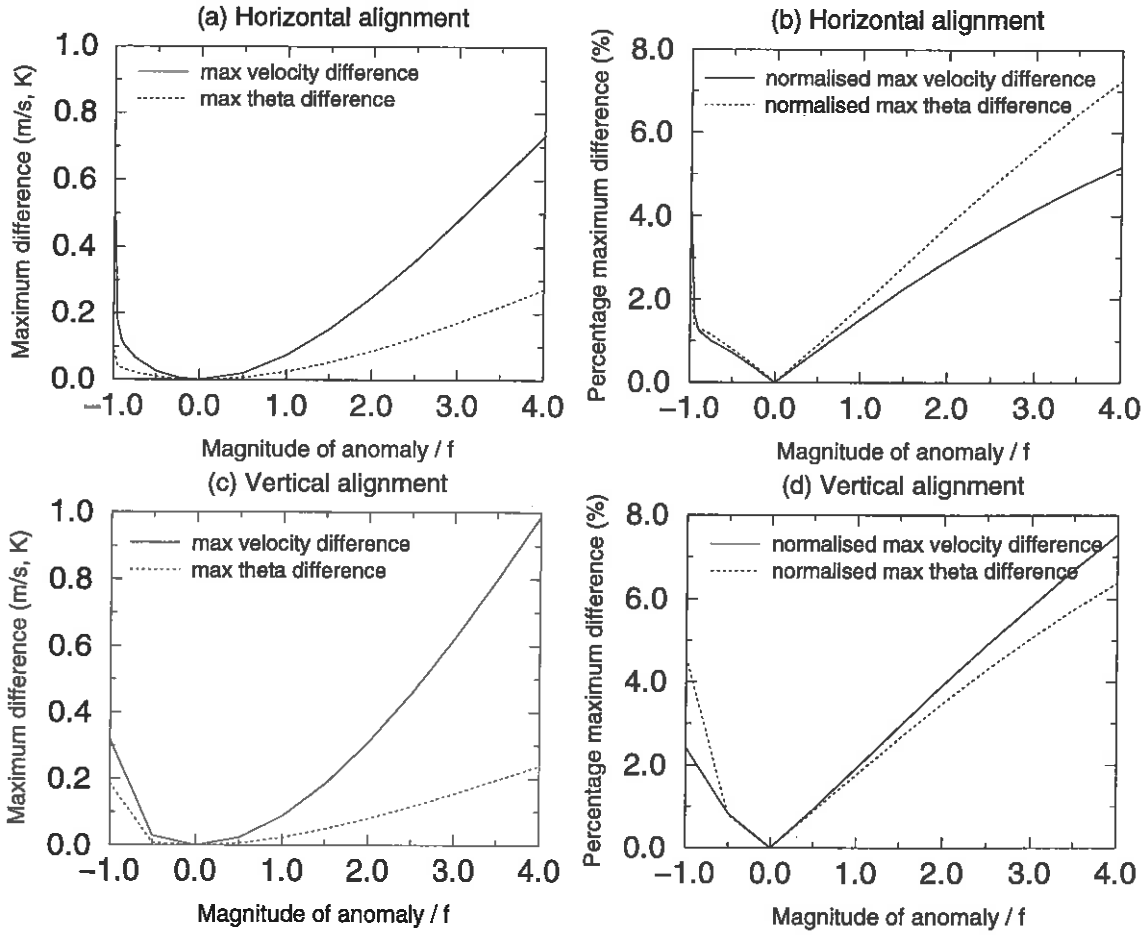

Fig 4: a) and c) Maximum, b) and d) percentage maximum, difference between linearly superposed and full solutions for different magnitude anomalies whose centres are 1.5 diameters apart, a) and b) horizontally, c) and d) vertically.

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